OPTIMUM TOLERANCE ALLOCATION AND PROCESS SELECTION USING SIMULATED ANNEALING

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Abstract An important consideration in product design is the assignment of tolerances to individual component dimensions so that the product can be produced economically and function properly. This work presents a procedure to allocate tolerance among functional dimensions to ensure minimum manufacturing cost while satisfying the functional requirements. The objective of the present work is to allocate tolerances and to select suitable processes among alternatives to minimize manufacturing cost. Simulated Annealing (SA) procedure is used for minimizing the total manufacturing cost of a part or assembly while meeting all critical and functional constraints imposed upon the design. Experimental results are tabulated.

Keywords: *Tolerance Allocation; Tolerance Optimization; Process Selection; Simulated Annealing.*

INTRODUCTION

Tolerancing is one of the most important task in product and manufacturing process design. The techniques of tolerancing are developed continuously over the years due to increasing demand of quality products and to satisfy the customers' requirement. The allocation of tolerances has a close relationship with the manufacturing cost and the functionality of the assembly. Tight manufacturing tolerance results in excessive process cost, while loose tolerance may lead to increase in waste and assembly problems. In tolerance allocation procedure, the assembly tolerance is distributed among individual components in such a way that the functional requirements are met with minimum total manufacturing cost.

In the early phase of design, the assembly tolerance is identified based on functional requirements. Further, component tolerances are determined based on the above assembly tolerance. Typically, tolerance allocation has been based on the experience of the designer. However resulting tolerances usually do not ensure the minimum production cost. For example, in a piston and cylinder assembly, the design specification is the clearance between piston and cylinder bore. The functional requirement of the above assembly is met by specifying the clearance between piston and cylinder bore. The tolerances on the piston and cylinder bore affect the functionality of the assembly. Lee et al., [1990] and Chase and Parkinson [1991] have reported that the investigations pertaining to tolerance synthesis are focussed mainly on two areas: (i) minimization of direct manufacturing cost and (ii) minimization of sensitivity of tolerances to the variations in the manufacturing processes.

TOLERANCE ALLOCATION

Tolerance Cost

An important consideration in product design is the assignment of tolerances to individual component dimensions so that the product can be produced economically and function properly [Chase et al., 1995]. The assembly tolerance is known or can be determined from the functional requirements, and the component tolerances are decided based on process capabilities of the production processes.

Costs to produce tolerances are generally available. The specification of tolerances on the parts determines the cost of manufacturing. The problem is to allocate tolerance to reduce the cost. When alternate processes are available for various processes, the problem becomes more complex and requires process selection also.

Assembly Tolerance Stack-up

In a mechanical assembly, when component tolerances are accumulated it may affect functionality of the assembly, which is known as *assembly tolerance stack-up*. Independent dimensions form the assembly tolerance loop and the loop is expressed generally as

$$Y = g(X_1, X_2, X_3, \ldots, X_N) \quad (1)$$

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The two most common methods used to find tolerance stack are: Worst Case (WC) and Root Sum Squares (RSS) method.

The worst-case tolerance models considered are given below.

For linear assembly

$$T_a = \sum t_i \qquad (2)$$

For non-linear assembly

$$T_a = \sum \left[\frac{\delta g}{\delta X_i} \right] t_i \qquad (3)$$

and the statistical tolerance stack models are:

For linear assembly

$$T_a = \sqrt{\sum T_i^{\ 2}} \qquad (4)$$

For non-linear assembly

$$T_a = \sum \left[\frac{\delta g}{\delta X_i} \right]^2 t_i^{\ 2} \qquad (5)$$

Where

T_a = Assembly tolerance

t_i = Feature tolerance of i^{th} component

$\left[\frac{\delta g}{\delta X_i} \right]$ = Sensitivity of the component dimension '*i*' to assembly variation '*g*'

In this work, statistical stack-up model is used for tolerance allocation, which consider that the process variations are always following a normal distribution.

Lee and Woo [1993] have combined tolerance allocation with process selection using a 'Branch and Bound' search procedure. They have selected a problem having 1.5 million combinations and reduced the search using Branch and Bound search to just 2554. The disadvantage of such procedure is computational time. SA algorithm is a powerful heuristic search technique employed for various optimization problems. An attempt has been made to employ SA procedure for tolerance allocation problem discussed in the literature [Choi, R. H. et al., 2000].

Several researchers working in the area of tolerance allocation have used different tolerance-cost functions such as linear, reciprocal, reciprocal square, exponential, etc., for defining the relationship. Although other functions can also define the relationship, the performance of these functions is highly dependent on field data [Choi, R. H. et al., 2000]. In the present work a reciprocal tolerance-cost function is employed (Eq. 6)

$$C(t) = a + \frac{b}{t} \qquad (6)$$

Where

a = Fixed cost

b = Cost of producing a single piece dimension to a specified tolerance.

When more than one process is available to produce a dimension to a specified tolerance, the relationship between tolerance, cost and process is to be established.

Tolerance-cost Model

Since the tolerance-cost values are known only at discrete points, a set of binary co-efficient having zero or one, are used to turn on or off some of the processes such that for each part only one cost value is permitted during cost evaluation [Balas, 1965]. When the sum of the process tolerances exceed the assembly tolerance limit, the particular combination of processes become infeasible.

The minimum cost function is expressed as

Minimize, $$\text{Cost} = \sum_{i=1}^{N} \sum_{j=1}^{m_i} B_{ij} C_{ij} \qquad (7)$$

Subjected to $$T_a^{\ 2} = \sum_{i=1}^{N} \sum_{j=1}^{m_i} B_{ij} t^2_{\ ij} \qquad (8)$$

and to ensure only one process per component,

$$\sum_{j=1}^{m_i} B_{ij} = 1 \ (i = 1, 2, 3, \ldots, N)$$

Where C_{ij} = Cost of producing '*i*'th component to a particular tolerance using '*j*' th process

B_{ij} = 0 or 1

SIMULATED ANNEALING

The simulated annealing algorithm was derived from statistical mechanism proposed by Kirkpatrick [1983]. It is based on the analogy between the annealing of solids and the problem of solving combinatorial optimization problems. A generic procedure of SA based on Parthasaraty and Rajendran [1998] is given below.

Step 1: Get an initial solution, *S*.

Step 2: Set an initial temperature, *T*>0.

Step 3: While not frozen do the following:

Step 3.1: Do the following *n* times:

Step 3.1.1: Sample a neighbour *S*' from *S*.

Step 3.1.2: Let Δ = cost (*S*') − cost (*S*).

Step 3.1.3: If Δ <= 0

then set *S* = *S*' (downhill move)

else set *S* = *S*' with a probability, *p*.

(*p* = exp. (−Δ / *T*))

Step 3.2: Set *T* = *r* x *T*,

where *r* is the reduction factor.

Step 4: Return *S*.

It consists of a set of iterations. Initial solution is generated randomly and is considered as a seed solution. A neighbourhood solution is generated from the current solution by a *perturbation mechanism*. The neighbourhood is defined by choice of the perturbation mechanism.

Once the new solution (neighbourhood) is created, it is compared with the seed for any improvement in the cost function. If the change in the cost function is negative, the perturbed solution is directly taken as *seed.* Otherwise, it is accepted as seed according to Metropolis's criterion [Metropolis et. al., 1953] based on Boltzman's probability. According to Metropolis's criterion, if the difference between the cost function (*Z*) values of the current and the perturbed solutions is equal to or larger than zero, a random number δ in the range of [0,1] is generated from a uniform distribution. If exp. $(-\Delta / T) > \delta$ then the perturbed solution is accepted as seed solution. If not, the current solution is unchanged. In this equation, 'Δ' is the difference between the cost function values of the two solutions.

PROPOSED PROCEDURE

Simulated Annealing algorithm is adopted suitably for the tolerance allocation problem under study. Simulated Annealing starts with a randomly generated seed value, which contains four parameters representing component tolerances i.e., X_1, X_2, X_3 and X_4. The feasiblity of seed is checked before evaluating its cost. Each evaluation is only a summation of current manufacturing cost. The SA parameter settings proposed by Parthasaraty and Rajendran [1998] is adopted in the present work. They are:

$T_i = 500$, $T_f = 20$, $n = 50$ and $r = 0.9$

Step 1: Generate a feasible seed '*S*' randomly.

Step 2: Repet steps 3 to 8 until $T<T_f$.

Step 3: Evaluate the cost of seed *C(S).* Cost is evaluated based on the tolerance values of the four parameters and the respective processes by referring to the tolerance-cost data given in Table 1.

Step 4: Repeat steps 5 to 7, *n* times

Step 5: Generate a feasible neighbour *S'* in the neighbourhood region of *S* by using peturbation mechanism and evaluate the same. In the present work, an adjacent-pairwise interchange mechanism is employed to generate a neighbour. The neighbourhood size is defined by the length of the binary string representing tolerance parameters.

Step 6: Evaluate the cost difference .

$\Delta = C(S') - C(S)$

Step 7: If Δ is negative,
accept neighbour as seed and
goto step 5
else
accept the inferior neighbour as seed with a probability $p = \exp(-\Delta/T)$ and goto step 5.

Step 8: Set $T = r * T$, and goto step 4

Step 9: Return the parameters, corresponding processes and cost represented by the seed.

An overrunning clutch assembly shown in Fig.1 having four components considered in this paper is adopted from Greenwood and Chase [1988]. The cage and hub are manufactured with three processes each and the balls are manufactured with two processes each, so that there are 36 (3×2×2×3) different combinations of processes available for manufacturing the overrunning clutch assembly.

Fig. 2 indicates the number of processes used each component with tolerance cost curves for overrunning clutch assembly. When number of components is more in an assembly, the search space becomes large and a heuristic procedure such as SA algorithm can be used to identify a combination, which gives minimum cost. In general, if the assembly has *N* parts and each part can be manufactured using *m* processes, then the number combinations of processes is $m_1 \times m_2 \times m_3 \times \ldots \times m_N$.

Where
N = Number of parts in the assembly
m_i = Number of alternate process for the component 'i'

ILLUSTRATIVE EXAMPLE

The overrunning clutch assembly problem considered by Fortini [1967] was re-examined by Greenwood and Chase [1988], Krishnaswami and Mayne [1994], Chang and Kusiak [1997] and Choi, R. H. et. al, [2000] for tolerance allocation with cost minimization as the objective. The contact angle *Y* is the functional dimension that must be controlled within tolerance stack-up limit. The contact angle (design function) is expressed as

$$Y = g(X_1, X_2, X_3, X_4) = \cos^{-1}\left[\frac{X_1 + \frac{(X_2 + X_3)}{2}}{X_4 - \frac{(X_2 + X_3)}{2}}\right] \quad (9)$$

The design function value is given by 0.122 ± 0.035 rad (7.0 ± 2.0°). Based on the above discussions, a simple SA procedure is employed for tolerance allocation and process selection with an objective of cost minimization.

The manufacturing cost of the assembly is given by

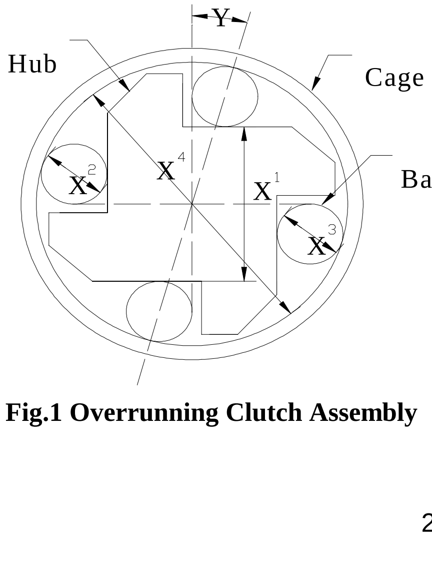

Fig.1 Overrunning Clutch Assembly

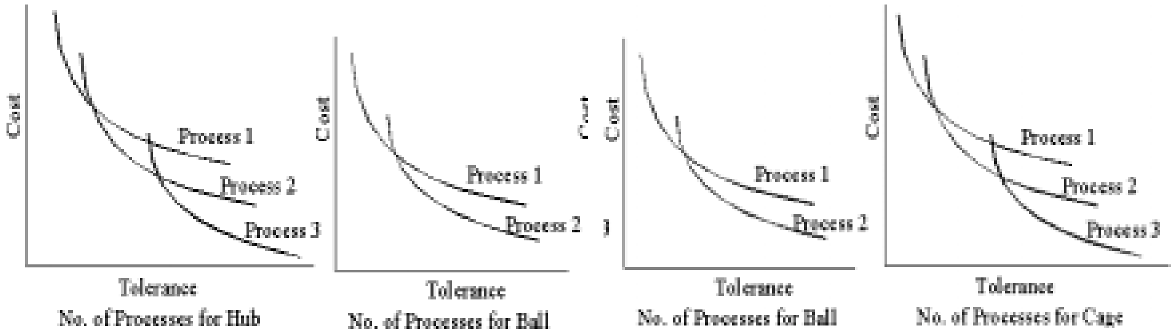

Fig 2. Processes Available for Overrunning Clutch Assembly

Minimize, $\text{Cost} = \sum_{i=1}^{N}\sum_{j=1}^{m_i} B_{ij}\left[a_{ij} + \frac{b_{ij}}{t_{ij}}\right]$ (10)

Subjected to $\sum_{i=1}^{4}\sum_{j=1}^{m_i}\left[\frac{\delta Y}{\delta X_i}\right]^2 B_{ij} t^2_{ij} \le 0.035^2$

$lt_{ij} \le t_{ij} \le ut_{ij}$

and $\sum_{j=1}^{m_i} B_{ij} = 1$ (i = 1, 2, 3, . . . , N)

Where B_{ij} = 0 or 1. (i = 1, 2, 3, 4)

(j = 1, 2, 3, . . . , m_i)

$\left[\frac{\delta Y}{\delta X_1}\right] = -0.1039$, $\left[\frac{\delta Y}{\delta X_2}\right] = -0.1035$,

$\left[\frac{\delta Y}{\delta X_3}\right] = -0.1035$ and $\left[\frac{\delta Y}{\delta X_4}\right] = 0.1032$

lt = Lower limit of tolerance

ut = Upper limit of tolerance

Table 1. shows the cost data for each alternate process and the range of tolerances for each dimension respectively. The result obtained using proposed procedure is shown in Table 2. Choi, R. H. et al [2000] has adopted an exhaustive search procedure in which all possible combinations of alternate processes are considered for process selection. The disadvantage of this method is the amount of computation required especially when the total number of combinations is large. In this paper, a simple SA procedure is employed for tolerance allocation and process selection with cost minimization as the objective. In this study zero-one algorithm works with SA for selecting the appropriate process. Cost obtained by SA is 1.95% higher than the result reported. But SA can be used effectively when the total number of process combinations is large.

SUMMARY

Tolerance allocation to a set of dimensions and corresponding combination of processes among alternative processes is done using a simple meta-heuristic search procedure with an objective of cost minimization. As described earlier, the major advantage of SA algorithm is, it can be effectively used when the total number of process combinations is large. With suitable memory models, alternative process combinations giving same cost can also be obtained using SA, if available. This is useful for a decision-maker to select a solution among a set of solutions.

Table 1 Tolerance and cost data for the overrunning clutch assembly

Dimension mm	Tolerance	X_1 = 55.29		X_2 = 22.86		X_3 = 22.86		X_4 = 101.69	
Process	Limits	Tolerance mm	Cost data	Tolerance mm	Cost data	Tolerance mm	Cost data	Tolerance mm	Cost data
Process 1	Lower (lt)	0.015	a = 10.0 b = 0.015	0.02	a = 8.00 b = 0.25	0.04	a = 2.5 b = 0.3	0.08	A = 4.00 b = 0.56
	Upper (ut)	0.08		0.15		0.2		0.12	
Process 2	Lower (lt)	0.06	a = 5.00 b = 0.50	0.08	a = 3.00 b = 0.65	0.12	a = 5.0 b = 0.045	0.15	A = 6.00 b = 0.16
	Upper (ut)	0.15		0.30		0.25		0.25	
Process 3	Lower (lt)	0.12	a = 3.50 b = 0.75	NA	NA	NA	NA	0.2	A = 0.50 b = 0.88
	Upper (ut)	0.25		NA		NA		0.4	

Table 2 Results obtained using SA

Nominal Size	Process Selected and Tolerance Assigned by Choi, R. H. et. al, (2000)		Process Selected and Tolerance Assigned by Simulated Annealing	
Nominal Size (mm)	Process Selected	Tolerance (mm)	Process Selected	Tolerance (mm)
$X_1 = 55.29$	3	± 0.179806	2	± 0.147283
$X_2 = 22.86$	2	± 0.165358	2	± 0.141067
$X_3 = 22.86$	1	± 0.120132	1	± 0.169607
$X_4 = 101.69$	3	± 0.200581	3	± 0.209358
Total Cost $		24.486553		24.974683

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